Echelon Form and Systems of Equations

July 26, 2021

Question 1

a) Give an example of a linear system with a 0 row but no free variables
b) Give an example of a linear system with a free variable but no 0-row.
c) Give an example of a linear system with a unique solution.
d) Give an example of a linear system with no solutions.

Question 2
Put the following matrices into Echelon Form and find the solution set of each.

a) $\begin{pmatrix} 1 & 1 & | & 2 \\ 1 & -3 & | & 4 \end{pmatrix}$

b) $\begin{pmatrix} 1 & 1 & -1 & | & 2 \\ 4 & 8 & -12 & | & 8 \\ 0 & 6 & -1 & | & 11 \end{pmatrix}$

c) $\begin{pmatrix} 1 & 1 & 0 & | & -1 \\ 0 & 1 & -1 & | & -1 \\ 0 & 4 & -4 & | & -8 \end{pmatrix}$

Question 3
Write the coefficient matrix for the following systems. In each case, state if the matrix is in echelon form.
a)

$$x_1 + 2x_2 - 3x_3 = 5$$
$$x_2 - 100x_3 = 1$$

b)

$$x_1 - x_2 + x_3 = 0$$
$$x_2 + x_3 = 0$$
$$x_1 \qquad\qquad = 0$$

c)

$$x_1 - 4x_2 = 10$$
$$x_2 = 8$$
$$x_1 - 6x_2 = 2$$

c)

$$x_1 + x_2 + x_3 + x_4 = 6$$
$$x_2 \qquad - x_4 = 2$$
$$x_3 \qquad = 6$$

Challenge- Extra Credit Question

Can you exhibit a linear system with 3 equations and 2 unknowns (x_1, x_2)

$$a_{11}x_1 + a_{12}x_2 = b_1$$
$$a_{21}x_1 + a_{22}x_2 = b_2$$
$$a_{31}x_1 + a_{32}x_2 = b_3$$

that has a solution for any values $b_1.b_2, b_3$?